

# The effect of photo-electric absorption on space-charge limited flow in pulsars

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## ABSTRACT

Photo-electric absorption of blackbody photons is an important process which limits the acceleration of ions under the space-charge limited flow boundary condition at the polar caps of pulsars with positive corotational charge density. Photo-electric cross-sections in high magnetic fields have been found for the geometrical conditions of the problem, and ion transition rates calculated as functions of the surface temperatures on both the polar cap and the general neutron-star surface. The general surface temperature is the more important and, unless it is below  $10^5$  K, limits the acceleration electric field in the open magnetosphere to values far below those needed either for electron-positron pair creation or slot-gap X-ray sources. But such ion beams are unstable against growth of a quasi-longitudinal Langmuir mode at rates that can be observationally significant as a source of coherent radio emission.

## Key words:

pulsars: general - stars: neutron - plasma - instabilities

## 1 INTRODUCTION

Two previous papers have given the results of a study of reverse-electron flow in isolated neutron stars with positive polar-cap corotational charge density  $\rho_{GJ}$ , in which free emission of ions maintains the boundary condition for space-charge limited flow. The sign of this quantity depends on the relation between rotation spin  $\Omega$  and polar magnetic flux density  $\mathbf{B}$ . We refer to Goldreich & Julian (1969) for its Euclidean-space definition, and to Muslimov & Tsygan (1992) and Muslimov & Harding (1997) for the general-relativistic modification used here. Most publications have assumed the parallel case,  $\Omega \cdot \mathbf{B} > 0$ , with  $\rho_{GJ} < 0$  and electron acceleration. But there does not appear to be any reason why the antiparallel case with  $\rho_{GJ} > 0$  and ion and positron acceleration should not be present in the neutron-star population. The two earlier papers (Jones 2010a, 2011; hereafter Papers I and II) considered in some detail the consequences of proton formation in the electron-positron showers produced by the reverse-electron flux at the polar cap and its relation with observed radio-pulsar phenomena. But an important omission was the effect of reverse-electron flow on the electrostatics of particle acceleration under the space-charge limited boundary condition, and this is the subject of the present paper.

In this paper, the subscripts  $\parallel$  and  $\perp$  are with reference to the local magnetic field direction. Under the surface

electric-field boundary condition  $\mathbf{E}_{\parallel} \neq 0$  assumed in the classic paper of Ruderman & Sutherland (1975), reverse-electron flow does not modify ion or positron acceleration in any important way because the surface value of the acceleration field  $\mathbf{E}_{\parallel}$  is unconstrained. However, the presence of a reverse-electron negative charge density under the surface boundary condition  $\mathbf{E}_{\parallel} = 0$  of space-charge limited flow can, in certain cases, reduce the acceleration field to values so low that there is no possibility of electron-positron pair creation within the open magnetosphere above the polar cap or of a slot-gap X-ray source. Photo-electric absorption of polar-cap and neutron-star surface blackbody photons by accelerated ions is the source of the reverse-electron flux and the process is analogous with electron-positron pair creation in its effect on the acceleration field. Heating of the polar-cap surface by a reverse flux of photo-electrons has been considered previously by Cheng & Ruderman (1977), also by Muslimov & Tsygan (1987) in relation to an instability of a liquid surface under the boundary condition  $\mathbf{E}_{\parallel} \neq 0$ . The present paper is concerned not with heating effects but with the electrostatic consequences of the negative charge density that reverse-electron flow implies.

There have been many calculations of the photo-electric cross-section for atomic hydrogen in the high magnetic fields and temperatures of model neutron-star atmospheres. We refer to Potekhin & Pavlov (1997) for a critical survey of this work and to Potekhin, Pavlov & Ventura (1997) for a study of the adiabatic approximation used by many authors. Although model atmospheres containing low- $Z$  ele-

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ments have been constructed (see, for example, Mori & Ho 2007), there appear to be no published explicit photo-electric cross-sections for such elements in high magnetic fields. Also, the intervals of electron separation and photon energies relevant to the accelerated ions in the present work differ from those of neutron-star atmospheres. But the requirements of this paper have some simple features compared with the latter problem. It is not necessary to consider the complexities of thermal ion motion transverse to the field and, in particular, the photon momentum component  $k_\perp$  is such that in relation to the electron cyclotron radius  $r_B$ , the condition  $k_\perp r_B \ll 1$  is always satisfied. Hence simple expressions, valid only in this limit, have been derived and are given in Appendix A. The zero-field cross-sections have been computed extensively (see, for example, the work of Reilman & Manson 1979) and in approximations much superior to those of Appendix A, but nevertheless we show that use of the high-magnetic field expressions is essential in this paper.

The region of open magnetosphere above the polar cap is assumed to have a circular cross-section of radius  $u_0(z)$ , where  $z + R$  is the coordinate on the magnetic axis and  $R$  is the neutron-star radius. The acceleration field has two components. The first, present at very low altitudes  $z \ll u_0(0)$  above the polar-cap surface, is the inertially-generated field described by Michel (1974). The second, and much the more important, contribution is produced by the Lense-Thirring effect modification to the corotational charge density first recognized by Muslimov & Tsygan (1992) and extends over moderately large distances  $z \sim R$ . The computational results of this paper are contained in Table 1, which gives the charge evolution of typical ions as they are accelerated.

The photon source is modelled as a polar cap of radius  $u_0(0)$  and proper-frame temperature  $T_{pc}$  embedded in a spherical neutron-star surface of uniform temperature  $T_s$ . We find that the latter temperature is much the more significant but, unfortunately, it is essentially unknown in the case of the general isolated neutron-star population. Blackbody emission has been detected reliably in only a small number of pulsars with ages less than  $10^6$  yr (see the review of Yakovlev & Pethick 2004) and even then there remains the problem of deciding whether the source is the polar cap or the whole surface.

Section 2 considers the simplest possible acceleration process: one that is time-independent and in a common state of plasma composition over the whole polar cap. It is shown that, in this case, acceleration is inadequate to produce secondary electron-positron cascades except for values of  $T_s$  that are too small to be observable. The relation between this result and earlier work on space-charge limited flow in the  $\mathbf{\Omega} \cdot \mathbf{B} < 0$  case (Papers I and II) is discussed in section 3. The flux of ions that are relativistic but not ultra-relativistic is unstable against growth of a quasi-longitudinal Langmuir mode at rates that appear large enough to produce Langmuir solitons and strong turbulence (Weatherall 1997, 1998; Melrose 2000; Melikidze Gil & Pataraya 2000; Asseo & Porzio 2006).

## 2 ACCELERATION AND PHOTO-ELECTRIC ABSORPTION

Solutions for the charge density  $\rho$ , for  $\rho_{GJ}$  and the electrostatic potential  $\Phi$  have been obtained for space-charge limited flow in a dipole field by Muslimov & Harding (1997) and by Harding & Muslimov (2001). These authors considered electrons, for which the inertial component of the acceleration field is negligible. For ion acceleration, the inertial component, though less important than the Lense-Thirring induced term should be included. The expressions given by Muslimov & Harding are complicated and therefore we shall make use of the boundary condition and geometrical form of the open flux-line magnetosphere to obtain a very simple approximate expression for  $\Phi$  which is adequate for the purposes of this paper. We calculate the photo-electric transition rate for an ion accelerated to Lorentz factor  $\gamma$  at altitude  $z$  above the magnetic pole.

There is little information about electron separation energies for multiply ionized atoms in the magnetic fields,  $10^{12} < B < 10^{13}$  G of interest here. The most recent calculations (Medin & Lai 2006) are restricted to atomic numbers  $Z = 6$  and 26 and further, in the latter case to  $B \geq 10^{14}$  G. We therefore adopt the expression used previously (Jones 1981) for the separation energy  $\epsilon_l$ ,

$$\epsilon_l = 0.35 B_{12}^{1/2} \frac{Z - l}{\sqrt{2l + 1}} \quad \text{keV}, \quad (1)$$

in which  $l = 0, 1, 2, \dots$  is the electron spatial degeneracy quantum number. When summed over all  $l$ , equation (1) gives agreement with the directly computed total energies of atoms with atomic numbers  $Z = 10$  and 20 (see Jones 1985) that is adequate for the work of this paper.

The inertial component of the acceleration field can be found at small  $z \ll u_0(0)$  by means of a one-dimensional approximation to the Poisson equation which neglects the boundary condition on the cylindrical surface  $u = u_0$  separating open and closed flux lines,

$$E_\parallel \approx E_i = \left( \frac{8\pi\rho_{GJ}Mc^2}{\tilde{Z}e} \right)^{1/2}, \quad (2)$$

for ions of mass  $M$  and charge  $\tilde{Z}$ . The field increases from the boundary condition  $\mathbf{E}_\parallel = 0$  to this constant value in a very small distance

$$z_i \approx \left( \frac{Mc^2}{18\pi\rho_{GJ}\tilde{Z}e} \right)^{1/2}. \quad (3)$$

Photo-electric absorption of blackbody photons modifies  $E_\parallel$  by introducing the negative charge density of the electrons accelerated toward the polar-cap surface. To estimate their effect, we can assume that the transitions all occur at a unique altitude  $z = h$ , increase the ion charge from  $\tilde{Z}$  to  $Z_h$ , and reduce the field to  $E_\parallel = 0$  at  $z = h$ . Then,

$$h \approx \frac{3}{2} z_i \frac{\tilde{Z}}{Z_h - \tilde{Z}}, \quad (4)$$

and the ion Lorentz factor at  $z = h$  is,

$$\gamma_h \approx 1 + \frac{1}{2} \frac{\tilde{Z}}{Z_h - \tilde{Z}}. \quad (5)$$

The electron chemical potential in the upper, low-density,

regions of the atmosphere at the polar cap surface is,

$$\mu_e = k_B T \ln \left( \pi r_B^2 N_e \left( \frac{2\pi\hbar^2}{mk_B T} \right)^{1/2} \right), \quad (6)$$

at electron number density  $N_e$ . We have estimated  $\tilde{Z}$  by comparing the value of  $\mu_e$  on the surface of last scattering in the atmosphere prior to acceleration with the  $\epsilon_l$  given by equation (1). On this basis, and because the dependence of  $\mu_e$  on  $B$  is only logarithmic, we have adopted the condition  $\epsilon_l = 26k_B T$  to obtain the  $\tilde{Z}$  used in Table 1. The polar-cap radius is that given by Harding & Muslimov (2001),

$$u_0(0) = \left( \frac{2\pi R^3}{cP f(1)} \right)^{1/2}, \quad (7)$$

in which  $P$  is the rotation period and  $f(1) = 1.368$  for a neutron star of mass  $1.4M_\odot$  and radius  $R = 1.2 \times 10^6$  cm.

The inertial component of  $E_\parallel$  ceases to be significant at  $h \sim 10^3$  cm. In principle, calculation of the electric potential  $\Phi$  at  $z > h$  requires the solutions given by Harding and Muslimov. But at  $z \gg u_0(0)$  a very simple expression can be found on the basis that the open flux lines are contained within a narrow cylindrical tube whose cross-sectional area increases only slowly with  $z$ . Given the boundary condition  $\Phi = 0$  on the surface  $u = u_0$  and that the radial electric field is  $E_\perp \gg E_\parallel$ , application of Gauss's theorem to a section of the tube at altitude  $z$  gives the approximation,

$$\Phi = \pi (u_0^2(z) - u^2) (\rho(z) - \rho_{GJ}(z)), \quad (8)$$

in terms of the difference between the charge density  $\rho(z)$  and the Goldreich-Julian charge density  $\rho_{GJ}(z)$ . We refer to Harding & Muslimov (2001) for this quantity at polar coordinate  $r = \eta R$  and adopt the expression,

$$\rho - \rho_{GJ} = \frac{\kappa B f(\eta)}{cP \alpha \eta^3 f(1)} \left( 1 - \frac{1}{\eta^3} \right) \cos \psi, \quad (9)$$

in which we have neglected a second term containing  $\sin \psi$  which is small at altitudes  $\eta$  of order unity. Equation (9) assumes a dipole field with surface value  $B$ . Here,  $\kappa$  is the dimensionless Lense-Thirring factor and  $\psi$  is the angle between  $\boldsymbol{\Omega}$  and  $\mathbf{B}$ . This expression is a satisfactory approximation at  $z > u_0(0)$  and  $z < 3R$ . At these altitudes, the function  $f(\eta)/\alpha f(1)$  including the red-shift factor  $\alpha$  can be well-approximated by unity and is a slowly varying function of  $\eta$ .

Equations (8) and (9) define the acceleration of an ion and give the relation between the altitude  $\eta$  and the Lorentz factor  $\gamma$  and we restrict transition rate calculations to points on the magnetic axis ( $u = 0$ ). The neutron-star surface is assumed to be Lambertian. Then unit element of surface area at a point which subtends an angle  $\zeta$  with the magnetic polar axis produces a photon number flux,

$$J_0(\omega_0, \theta) = \frac{\omega_0^2 n(\omega_0)}{4\pi^3 c^2 R^2} \frac{\cos(\zeta + \theta)}{1 + \eta^2 - 2\eta \cos \zeta}, \quad (10)$$

at altitude  $z = r - R$  on the axis, in which the photon occupation number is  $n(\omega_0)$  for photons of angular frequency  $\omega_0$ . The photon momenta from this element are at an angle  $\theta$  with the magnetic axis. Lorentz transformation parallel with  $\mathbf{B}$  to the rest frame of an ion on the axis moving outward with Lorentz factor  $\gamma$  gives the flux in that frame,

$$J(\omega, \chi) = \gamma J_0(\omega_0, \theta) (1 - \beta \cos \theta), \quad (11)$$

where  $\omega$  and  $\chi$  replace  $\omega_0$  and  $\theta$ , and  $\beta$  is the ion velocity in units of  $c$ . The photo-electric transition rate transformed back to the neutron-star frame at an altitude  $\eta - 1$  on the magnetic axis is,

$$\Gamma_l = \frac{1}{\gamma} \int_0^{\zeta_{max}} 2\pi R^2 \sin \zeta d\zeta \int_0^\infty d\omega J(\omega, \chi) \sigma(B, \omega, \chi), \quad (12)$$

in which the upper limit of integration is given by  $\cos \zeta_{max} = 1/\eta$  and the integral is in two sectors; the polar cap with temperature  $T_{pc}$  and radius given by equation (7), and the remaining visible neutron-star surface of temperature  $T_s$ . The cross-section is  $\sigma = (\sigma_0^a + \sigma_1^a + \sigma_1^b)/2$ . Here, the subscripts denote the Landau quantum number of the final electron. The photon polarization is denoted by the superscripts  $a, b$ , respectively perpendicular to or parallel with  $\mathbf{k} \times \mathbf{B}$ . General-relativistic corrections to photon propagation, including the red-shift factor, have been neglected here in equations (10) - (12). Equation (12) gives the transition rate as a function of the coordinate  $\eta$  on the magnetic axis and the Lorentz factor  $\gamma$  of the ion. Owing to the geometrical restriction of equation (12) to points on the magnetic axis, transition rates for ions accelerated off-axis at finite  $u(z)$  would be in error to a modest extent for the polar-cap contribution, but for the much more significant whole-surface term, the error is negligible. We refer to Appendix A for details of the individual cross-sections. Approximations and sources of error are also discussed there, as is the comparison with zero-field cross-sections.

The integration in equation (12) is over the polar cap with temperature  $T_{pc}$  and the remaining visible part of the neutron-star surface at temperature  $T_s$ . From the transition rates and the relations between  $\gamma$  and  $z$  given by equations (2) - (5), (8) and (9), two optical depths have been determined for three values each of nuclear charge and polar-cap magnetic flux density. These are,

$$\frac{1}{c} \int_0^h \Gamma_l(z) dz \quad (13)$$

which defines  $Z_h$  for a given  $\tilde{Z}$ . Its dependence on  $T_s$  is negligibly small principally because the factor  $\cos(\zeta + \theta) \approx 0$ . The second optical depth is,

$$\frac{1}{c} \int_h^{z_{max}} \Gamma_l(z) dz \quad (14)$$

which gives  $Z_\infty$  and is almost completely independent of  $T_{pc}$  owing to the unfavourable nature of the Lorentz transformation for polar-cap photons at  $z \gg u_0(0)$ . In both cases, integer charges are determined by whether or not the optical depth concerned is greater or less than unity. This is usually obvious because optical depths decrease rapidly with increasing ion charge and electron separation energy.

Equations (4) and (5) are based on the effect of reverse-electron negative charge density at  $z \ll u_0(0)$ . Photo-electric absorption at  $z \gg u_0(0)$  also introduces a reverse-electron flux and negative charge density in any interval of  $z$  in which there is a finite  $E_\parallel$ . Thus  $\rho u_0^2$  for ultra-relativistic ions is not constant but is a function of  $z$  determined by the local transition rate,

$$\frac{1}{\rho u_0^2} \frac{\partial(\rho u_0^2)}{\partial z} = \sum_l \frac{2\Gamma_l}{cZ(z)}.$$

and so reduces the acceleration field obtained from equation (8). There can be acceleration in any interval of  $z$  only if photo-electric rates are such that  $\partial\Phi/\partial z < 0$  and this condition leads at once to the inequality,

$$\sum_l \frac{2\Gamma_l}{cZ(z)} < \kappa \frac{\partial}{\partial z} \left( 1 - \frac{1}{\eta^3} \right), \quad (15)$$

in which the fractional error in the right-hand term is of order  $\kappa$  relative to unity, and  $Z(z)$  is the local value of the ion charge. Because the principal aim of this calculation is to see if there can be sufficient acceleration to produce electron-positron pair creation, we adopt the following procedure. We scale  $\Phi$  so as to give an ion Lorentz factor  $\gamma = 500$  at  $z_{max}$ , that is, a potential difference of about  $10^3$  GeV. Then the lowest ion charge that has less than unit optical depth in the interval  $h < z < z_{max}$  is defined as  $Z_\infty$ . But if this acceleration is to be possible, photo-electric rates must not exceed the limit defined above. On integrating (15), we find that  $Z_\infty$  must satisfy the approximate inequality  $Z_\infty - Z_h < \kappa Z_\infty/2$  for consistency. This condition is maintained by an equilibrium such that the acceleration field satisfying Poisson's equation adjusts to those values that give the ion acceleration and photo-electric transition rates necessary for self-consistency. To enlarge on this, we can note that if photo-ionization were complete at an altitude  $z_c$ , acceleration of the bare nucleus would continue at  $z > z_c$ , the potential difference remaining for this being approximately  $\Phi(z) - \Phi(z_c)$ . But integration of (15) for this case gives the approximate constraint,

$$Z - Z_h < \frac{\kappa Z}{2} \left( 1 - \frac{1}{\eta_c^3} \right),$$

so that values of  $Z - Z_h$  for which continued acceleration is possible are severely constrained. The equilibrium we have mentioned above means that the acceleration field given by Poisson's equation adjusts at all  $z$  to that required for photo-ionization transition rates no larger than those satisfying the inequality (15). Reference to Table 1 shows that continued acceleration over a potential difference approaching the maximum given by equations (8) and (9) is not possible except for low  $B$  and high values of  $T_{pc}$ .

The optical depth integration is continued to  $z_{max} = 3R$  although the major contribution is from much lower altitudes. But the local magnetic flux density is much reduced near  $z_{max}$  and therefore the separation energy used is the greater of that given by equation (1) or a zero-field expression given by a Rydberg formula in terms of the screened nuclear charge and the appropriate principal quantum number for the electron shell.

The Table shows that  $Z_h$  differs little from  $\tilde{Z}$  for any values of  $B$  and  $T_{pc}$ . There are two reasons for this. Firstly, the cross-section for transitions to the lowest Landau state, the only final electron state accessible at low values of  $\gamma$ , contains the factor  $\sin^2 \chi$ , which is always small. Thus use of the zero-field cross-section in this region would be quite incorrect. Secondly, as previously explained in relation to equations (4) and (5), the reverse-electron negative charge density itself limits acceleration at  $z < h$ . Most photo-electric absorption occurs at  $z \gg u_0(0)$ , is  $T_s$ -dependent, and consists of transitions to the  $n = 1$  Landau state whose cross-

sections contain  $\cos^2 \chi$ . These, of course, require acceleration to higher Lorentz factors.

The estimate of the Lense-Thirring parameter used by Muslimov & Harding (1997) is  $\kappa = 0.15$ . Values of  $Z_\infty$  are given for neutron-star surface temperature  $T_s = 1.0 \times 10^5$  K in column 6 and by reference to the integrated form of (15) show that  $Z_\infty - Z_h$  is such that acceleration is not always possible even at this low temperature. At  $2.0 \times 10^5$  K, acceleration occurs only in the limited number of cases in which high values of  $T_{pc}$  give  $Z_h = Z$ . Because  $Z_h$  differs little from  $\tilde{Z}$ , we can see that the basic uncertainty is in the latter quantity. Estimates more refined than our condition derived directly from equations (1) and (6) might give lower or higher values of  $\tilde{Z}$ . In the former case, values of  $Z_h$  would also be smaller, hence the possibility of significant acceleration would be reduced. In the latter,  $Z_\infty - Z_h$  would be reduced. However, if photo-electric ionization is complete at  $z = h$ , the energy flux of reverse electrons is very limited. Typically, for  $Z_h - \tilde{Z} = 1$ , equation (5) shows that the reverse electron energy input to the polar cap is no more than  $\sim 20$  GeV per ion, giving only low rates of pair creation by inverse Compton scattering or by  $(n, \gamma)$  reactions.

The values of polar-cap magnetic flux density considered in Table 1 were chosen so as to include the median and mean values of  $\log_{10} B$  for the Wang, Manchester & Johnston (2007) catalogue of pulsars that exhibit nulls. For fields much smaller than  $10^{12}$  G, the approximation  $k_\perp r_B \ll 1$  made in Appendix A fails, Landau level spacings become small, and the photo-electric cross-section approaches the zero-field value. Thus transition rates remain high. On the basis of Table 1, we find, with some confidence, that the Muslimov-Tsygan acceleration field is in general so reduced by electron production that electron-positron pair creation is not possible. In this state, which we assume to be time-independent, there is an equilibrium between the photo-electric transition rates and the ion acceleration needed to produce them. In many cases, ion Lorentz factors at  $z_{max}$  may differ little from  $\gamma_h$ . Our finding is limited, of course, to neutron stars whose whole-surface temperature is  $T_s \geq 10^5$  K. But a temperature of  $10^5$  K is problematical from all points of view. It is unobservable in a compact source given present techniques and the energy flux needed to maintain it is small and easily supplied by many possible dissipative processes. Also, measured photon spectra do not always allow the unambiguous separation of polar cap and whole-surface radiation.

### 3 CONCLUSIONS

The results described in the previous Section are a further example of the diverse forms of polar-cap acceleration under the space-charge limited flow boundary condition that exist for the  $\mathbf{\Omega} \cdot \mathbf{B} < 0$  case. It was assumed that the composition of the accelerated plasma is uniform over the whole polar-cap area. Before examining possible observable consequences of the uniform state, we need to mention briefly the impact of these results on the polar-cap acceleration which was considered at length in Paper II.

We first comment on the assumed stability of the state described in Section 2. In the physical processes concerned, there is no obvious characteristic time apart from ion flight

**Table 1.** For a range of values of polar-cap magnetic flux density, temperature and nuclear charge  $Z$ , values of the ion charge in local thermodynamic equilibrium immediately before acceleration ( $\tilde{Z}$ ) and at the end of the first stage of acceleration ( $Z_h$ ) are given. The ion charge at the end of acceleration  $Z_\infty$  is given in the right-hand four columns for arbitrarily selected values of the general surface temperature  $T_s$  subject to the explanation and qualifications discussed in Section 2. For  $T_s \geq 5.0 \times 10^5$  K, the ion charge is always  $Z_\infty = Z$ .

| $B$<br>$10^{12}\text{G}$ | $T_{pc}$<br>$10^6\text{K}$ | $Z$ | $\tilde{Z}$ | $Z_h$ | $Z_\infty$<br>$T_s = 0.1$ | $Z_\infty$<br>0.2 | $Z_\infty$<br>0.3 | $Z_\infty$<br>0.4 |
|--------------------------|----------------------------|-----|-------------|-------|---------------------------|-------------------|-------------------|-------------------|
| 1.0                      | 1.0                        | 10  | 9           | 9     | 10                        | 10                |                   |                   |
|                          |                            | 20  | 16          | 17    | 18                        | 20                |                   |                   |
|                          |                            | 26  | 20          | 21    | 24                        | 26                |                   |                   |
|                          | 1.5                        | 10  | 9           | 9     | 10                        | 10                |                   |                   |
|                          |                            | 20  | 18          | 20    | 20                        | 20                |                   |                   |
|                          |                            | 26  | 23          | 25    | 25                        | 26                |                   |                   |
|                          | 2.0                        | 10  | 10          | 10    | 10                        | 10                |                   |                   |
|                          |                            | 20  | 19          | 20    | 20                        | 20                |                   |                   |
|                          |                            | 26  | 24          | 25    | 25                        | 26                |                   |                   |
|                          | 3.0                        | 10  | 7           | 7     | 8                         | 10                |                   |                   |
|                          |                            | 20  | 13          | 14    | 15                        | 19                | 20                |                   |
|                          |                            | 26  | 16          | 17    | 18                        | 24                | 26                |                   |
| 10.0                     | 1.0                        | 10  | 8           | 9     | 9                         | 10                |                   |                   |
|                          |                            | 20  | 16          | 17    | 17                        | 19                | 20                |                   |
|                          |                            | 26  | 19          | 20    | 20                        | 24                | 26                |                   |
|                          | 1.5                        | 10  | 9           | 10    | 10                        | 10                |                   |                   |
|                          |                            | 20  | 17          | 18    | 18                        | 19                | 20                |                   |
|                          |                            | 26  | 21          | 23    | 23                        | 24                | 26                |                   |
|                          | 2.0                        | 10  | 5           | 5     | 6                         | 6                 | 9                 | 10                |
|                          |                            | 20  | 8           | 8     | 10                        | 11                | 18                | 19                |
|                          |                            | 26  | 11          | 11    | 12                        | 13                | 22                | 24                |
|                          | 1.5                        | 10  | 7           | 7     | 7                         | 7                 | 9                 | 10                |
|                          |                            | 20  | 12          | 12    | 14                        | 14                | 18                | 19                |
|                          |                            | 26  | 14          | 15    | 17                        | 17                | 22                | 24                |
|                          | 2.0                        | 10  | 8           | 8     | 9                         | 9                 | 9                 | 10                |
|                          |                            | 20  | 14          | 15    | 16                        | 16                | 18                | 19                |
|                          |                            | 26  | 17          | 18    | 20                        | 20                | 22                | 24                |

times which are of the order of  $Rc^{-1} \sim 4 \times 10^{-5}$  s. This is many orders of magnitude shorter than the proton diffusion time constant  $\tau_p$  defined in Paper II. Thus we do not believe that the possibility of instability arising from photo-electric absorption can materially change the conclusions of that work.

Decay of the giant-dipole resonance formed in reverse-electron showers produces protons, and it was shown in Papers I and II that this process leads to temporal instabilities in the composition of the plasma accelerated. It was proposed that the consequences of these include the phenomena of nulls and sub-pulse drift observed in some radio pulsars. The number of protons created per unit ion charge accelerated, defined as  $K$  in Papers I and II, is the significant parameter. It was shown in II that, in general, the state of the polar-cap must be time-dependent, with any given element of area alternating between the acceleration of ions and positrons or of protons. In such an element, proton acceleration produces no reverse-electron flux and so must be of limited duration. It was assumed in Paper II that acceleration would be through the full potential difference given here by equations (8) and (9), but this obviously needs some qualification. At any instant, there is positron and ion accel-

eration from only a fraction  $(K+1)^{-1}$  of the whole polar-cap area. Thus the photo-electric production of negative charge is confined to this fraction and for  $K \sim 5-10$  the charge density averaged over the cross-sectional area  $\pi u_0^2(z)$  does not differ much from that given by equation (9). Very roughly, the modified integrated form of (15),

$$Z_\infty - Z_h < \frac{1}{2}\kappa(K+1)Z_\infty, \quad (16)$$

is now the necessary condition for the existence of a non-zero acceleration potential difference. The actual value of  $Z_\infty - Z_h$  determines the amount by which the equilibrium acceleration potential is reduced from the full potential  $\Phi$ . Inspection of  $Z_\infty - Z_h$  values given in Table 1 shows that, for  $K \geq 5$  and with the exception of  $T_{pc} = 1.0 \times 10^6$  K, the acceleration potential is usually a large fraction of  $\Phi$ .

The uniform state considered here in Section 2 is of interest in connection with the usual assumption that secondary low-energy electron-positron pair creation is necessary for coherent radio emission. It has no obvious mechanism for electron-positron pair formation and the assumption might well be that, in an isolated neutron star, it would correspond with a long null state of radio and polar-cap X-ray emission.

However, the possibility that cold ions might be important in the growth of unstable quasi-longitudinal plasma modes was considered many years ago by Cheng & Ruderman (1980), but it appeared that the longitudinal effective mass  $M\gamma^3$  of the accelerated ions was too large to give the necessary growth rates. However, the small Lorentz factors found here prompt reconsideration because the longitudinal effective mass is much reduced although the energy flux of the ions remains high in comparison with typical pulsar radio luminosities.

In the uniform composition state, acceleration fields are everywhere small so that the reverse-electron energy flux gives values  $K < 1$  and a mixed plasma of ions and protons for which the instability arguments of Papers I and II are not necessarily valid. The plasma then has two cold components, ions and protons with small Lorentz factors  $\gamma_1$  and  $\gamma_2$ , respectively, with  $\gamma_1 \approx 0.5\gamma_2$ , and obviously satisfies the relativistic Penrose condition (see Buschauer & Benford 1977). But because the plasma is cold, we can easily find the growth rate of the quasi-longitudinal Langmuir mode studied by Asseo, Pelletier & Sol (1990) for which the dispersion relation is,

$$(\omega^2 - k_\parallel^2) \left( 1 - \frac{\omega_1^{*2}}{(\omega - k_\parallel\beta_1)^2} - \frac{\omega_2^{*2}}{(\omega - k_\parallel\beta_2)^2} \right) = k_\perp^2, \quad (17)$$

for angular frequency  $\omega$ , wave vector  $\mathbf{k}$ , and at ion and proton velocities  $\beta_1$  and  $\beta_2$ , respectively. The remaining quantities are  $\omega_i^* = \gamma_i^{-3/2}\omega_i$  in which,

$$\omega_i^2 = \frac{4\pi N_i q_i^2}{M_i}, \quad (18)$$

where  $N_i$ ,  $q_i$  and  $M_i$  are respectively the number density, charge and mass of each component  $i = 1, 2$ . The rest-frame plasma frequency for a given component is  $\gamma_i^{-1/2}\omega_i$  and therefore, following Asseo et al, we consider the case  $k_\perp = 0$  and  $k_\parallel = 2\omega_1^*\gamma_1^2$ , defining a new variable  $s$  through the relation,

$$\omega - k_\parallel\beta_1 = \omega_1^*(1 + s), \quad (19)$$

so that the dispersion relation becomes,

$$1 - \frac{1}{(s+1)^2} - \frac{C}{(s+\mu)^2} = 0 \quad (20)$$

in which  $\mu = \gamma_1^2/\gamma_2^2$  and  $C = \omega_2^{*2}/\omega_1^{*2}$ . Inspection of equation (20) shows that this quartic has two real roots which can easily be found numerically thus giving the remaining complex roots. In general, the growth rate found from the complex roots is of the order of  $10^{-1}\omega_1^*$ , but is smaller if another ion species with  $q_1/M_1 \neq q_2/M_2$  is substituted in place of the proton beam. Thus for a typical ion charge of  $q_1/e = 20$  and a proton beam of number density  $N_2 = 5N_1$  we can choose  $C = 0.045$  and  $\mu = 0.2$ , for which the amplitude growth rate is,

$$\text{Im}\omega = \pm 0.18\omega_1^* = \pm 4.0 \times 10^7 \left( \frac{B_{12}R^3}{\gamma_1^3 P r^3} \right)^{1/2} \quad (21)$$

rad s<sup>-1</sup>, at radius  $r$ , where  $B_{12}$  is the polar-cap magnetic flux density in units of  $10^{12}$  G and  $P$  is the rotation period. The amplitude growth factor, assuming constant  $\gamma_1$  the interval  $0 < z < z_{max}$ , is  $\exp \Lambda$ , where,

$$\begin{aligned} \Lambda &= \int_0^{z_{max}} dz \frac{\text{Im}\omega}{c} \\ &= 3.2 \times 10^3 \left( \frac{B_{12}}{\gamma_1^3 P} \right)^{1/2} \left( 1 - \left( \frac{R}{R + z_{max}} \right)^{1/2} \right). \end{aligned} \quad (22)$$

This is large for  $\gamma_1 < 10$ . Values of the mode frequency are relatively small,  $\omega \approx 2\gamma_1^2\omega_1^*$ , and may lie well below 400 MHz, but it would be interesting to see if evidence for it is present in pulsar spectra at low frequencies.

We have seen that the presence of ion and proton beams that are relativistic but not ultra-relativistic, previously unexpected in pulsars, can be associated with the growth of unstable quasi-longitudinal modes having growth rates high enough for the formation of Langmuir solitons and strong turbulence. We can refer to the review of Melrose (2000) and to the papers of Weatherall (1997) and (1998), Melikidze et al (2000) and Asseo & Porzio (2006) to note that the collapse of such solitons is widely thought to be a plausible source for the coherent radio emission. It is usually assumed that the instability occurs in secondary electron and positron beams with Lorentz factors of the order of  $10^2$  but it is the macroscopic fields of the solitons that lead to coherent emission independently of the particle type from which they are formed.

## ACKNOWLEDGMENTS

I thank the referee for directing my attention to the work of Potekhin, Pavlov & Ventura (1997) on the limitations of the adiabatic approximation for the photo-ionization of atomic hydrogen in very strong magnetic fields.

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## APPENDIX A: PHOTO-ELECTRIC ABSORPTION IN HIGH MAGNETIC FIELDS

The presence of a high magnetic field greatly modifies the final states that are accessible to the electron in photo-electric absorption. In particular, the Landau level spacing,  $\hbar\omega_B = 11.6$  keV at  $10^{12}$  G, is large compared with black-body photon energies. In the rest-frame of the accelerated ion, the photon momentum  $\mathbf{k}$  is almost parallel with  $\mathbf{B}$  and this much reduces transition rates to the  $n = 0$  Landau state. We adopt the Johnson & Lippmann (1949) spinor solutions for a free electron in a uniform magnetic field. These are satisfactory here because we require only the cross-section  $\sigma_0$  to the unique  $n = 0$  ground state, and the sum of the cross-sections to the two degenerate  $n = 1$  states. Photon polarization states are defined as those perpendicular to, or parallel with,  $\mathbf{k} \times \mathbf{B}$  and are denoted by superscripts  $(a, b)$  respectively. For the explicit form of the solutions and of the interaction with the electromagnetic field it will be convenient here to refer to Appendix A of Jones (2010b).

Although with later comment, we assume the adiabatic approximation for the partially-ionized ground state in which the electrons occupy single-particle states each with Landau quantum number  $n = 0$ . Recoil is neglected so that in the ion rest frame, the photo-electric thresholds are  $\epsilon_l + n\hbar\omega_B$ , where  $\epsilon_l$  is the separation energy given by equation (1) and  $n$  is here the electron final state Landau quantum number. Following equation (A6) of Jones (2010b), the transition matrix element from a photon with angular frequency  $\omega$  and polarization  $\epsilon^{a,b}$  to the  $n = 0$  ground state

with electron longitudinal momentum  $p$  is,

$$\left(\frac{2\pi\hbar e^2 c^2}{\omega}\right)^{1/2} \int r_{\perp} dr_{\perp} d\phi e^{i\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}} \Psi_{-1,0}^{\dagger}(p) \boldsymbol{\alpha} \cdot \boldsymbol{\epsilon}^{a,b} \Psi_{-1,0}(p - k_{\parallel}) G(p - k_{\parallel}) \quad (\text{A1})$$

in cylindrical polar coordinates, where  $G$  is the Fourier transform of the model bound electron state  $\sqrt{\kappa} \exp(-\kappa|z|)$ , in which  $\hbar^2 \kappa^2 = 2m\epsilon_l$ . The dependence of the cross-section on the charge of the ion is present solely through the Fourier transform. The model function, which replicates the correct asymptotic form of the true function, given in this Appendix by the parameter  $\kappa$ , is an adequate approximation for the purposes of this paper. The Johnson & Lippmann spinor for Landau number  $n \geq 0$  is,

$$\Psi_{-1,n}(p) = \begin{bmatrix} 0 \\ \psi_n \\ \frac{p_{\perp}}{2mc} \psi_{n-1} \\ -\frac{p_{\parallel}}{2mc} \psi_n \end{bmatrix},$$

where  $p_{\perp}^2 = 2mn\hbar\omega_B$ , and the normalized functions  $\psi_{nl}$  are the non-relativistic radial solutions of the Schrodinger equation for a free electron in a uniform magnetic field, in which  $l$  is the spatial degeneracy quantum number (see equations A3 - A5 of Jones 2010b). The azimuthal angle-dependence is  $\exp(-il(l-n)\phi)$ . The ion structure consists of spatially well-defined coaxial shells with sequential values  $l = 0, 1, 2, \dots$  and energies given by equation (1). The possibility of a hole state is not considered because its lifetime would be many orders of magnitude smaller than the photo-electric transition rates calculated here.

For the specific geometry of the photo-electric process considered here and for high fields  $B \geq 10^{12}$  G, the exponent in the matrix element, which is of the order of  $k_{\perp} r_B$  is always very small compared with unity and is set equal to zero. This introduces the selection rule  $\delta(n-l) = 0$  which leads to an easy evaluation of the transition rates. Hence, immediately from equation (A1), we obtain the cross-section,

$$\sigma_0^a = \frac{2\pi e^2 \sin^2 \chi}{cpm\omega} \left( \frac{\kappa^3 (2p - k_{\parallel})^2}{(\kappa^2 + (p - k_{\parallel})^2)^2} + \frac{\kappa^3 (2p + k_{\parallel})^2}{(\kappa^2 + (p + k_{\parallel})^2)^2} \right), \quad (\text{A2})$$

for transitions to the  $n = 0$  Landau state for polarization perpendicular to  $\mathbf{k} \times \mathbf{B}$ . In the parallel polarization case, the cross-section to the  $n = 0$  state is  $\sigma_0^b = 0$ . Proceeding in a similar way, the cross-sections to the two degenerate  $n = 1$  states are found to be,

$$\sigma_1^a = \frac{2\pi e^2 \cos^2 \chi}{cpm\omega} \left( \frac{\kappa^3 (p_{\perp}^2 + k_{\parallel}^2)}{(\kappa^2 + (p - k_{\parallel})^2)^2} + \frac{\kappa^3 (p_{\perp}^2 + k_{\parallel}^2)}{(\kappa^2 + (p + k_{\parallel})^2)^2} \right), \quad (\text{A3})$$

and  $\sigma_1^b = \sigma_1^a$ . It is to be emphasized again that these expressions are valid only under the condition that  $k_{\perp} r_B \ll 1$  which is valid for the photo-electric process considered here. These cross-sections, not too far above threshold and at  $10^{12}$  G, do not differ greatly from those given by the zero-field Kramers formula apart from the presence of the  $\sin^2 \chi$  factor in  $\sigma_0^a$ , and the zero parallel-polarization cross-section  $\sigma_0^b$ . These factors, within the adiabatic approximation, are a di-

rect consequence of the relative orientation of the photon polarization vector and the magnetic field. Transitions from  $n = 0$  to final states of  $n \geq 2$  have not been considered here because their matrix elements vanish in our approximation  $k_{\perp} r_{\perp} = 0$ .

The procedures used here are much inferior to those for zero-field cross-sections as in the work of Reilman & Manson (1979), in particular, the unperturbed final-state wave function and the state of the initial bound electron. The latter has been treated in the adiabatic approximation as a single-particle state with good quantum numbers  $n = 0$  and  $l$ . Use of this approximation in the case of atomic hydrogen has been extensively studied by Potekhin, Pavlov & Ventura (1997). The true ground state contains small components with  $n > 0$  which lead to a non-zero value of the cross-section  $\sigma_0^b$  which increases in significance as the threshold for the  $n = 1$  final state is approached. The same is true for any single-particle state in a multi-electron ion. But in multi-electron ions, there are further problems. Whilst a shell model is clearly the only useful basis for description of the atom, as in the zero-field case, there must be a residual electron-electron interaction whose matrix elements, though satisfying the  $\delta(n-l) = 0$  selection rule overall, can introduce an  $n = 1$  and  $l + 1$  component into the  $n = 0$  and  $l$  ground state and partially invalidate the adiabatic assumption. Relative to unity, the amplitude of such a component would be of order  $\Delta/\hbar\omega_B$ , where  $\Delta$  is the matrix element, and is almost certainly very small. But the component has a photo-electric transition matrix element to the  $n = 0$  final electron state that is not inhibited by the  $\sin \chi$  factor. Thus there must be a small non-zero value of  $\sigma_0^b$  and a correction to  $\sigma_0^a$ . Clearly the cross-sections obtained here are subject to some uncertainty for the many reasons that have been discussed. However, we do not need great accuracy in the present work because the photo-electric transition rates required are also dependent on the high-frequency exponential tail of the blackbody spectrum and on the Lorentz transformation to the ion rest frame. But it is important that cross-sections with the correct high-field Landau level structure and dependence on photon polarization be used.

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